Please check the examination details below before entering your candidate information

Candidate surname	Other names

Pearson Edexcel Level 3 GCE

Centre Number	Candidate Number

Thursday 16 May 2019

Afternoon | Paper Reference **8FM0-25**

Further Mathematics

Advanced Subsidiary
Further Mathematics options
25: Further Mechanics 1
(Part of options C, E, H and J)

You must have:
Mathematical Formulae and Statistical Tables (Green), calculator

Total Marks

Candidates may use any calculator allowed by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Unless otherwise indicated, whenever a value of g is required, take $g = 9.8 \text{ m s}^{-2}$ and give your answer to either 2 significant figures or 3 significant figures.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- The total mark for this part of the examination is 40. There are 4 questions.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over

1. A lorry of mass 16 000 kg moves along a straight horizontal road.

The lorry moves at a constant speed of 25 m s^{-1}

In an initial model for the motion of the lorry, the resistance to the motion of the lorry is modelled as having constant magnitude 16 000 N.

(a) Show that the engine of the lorry is working at a rate of 400 kW.

(4)

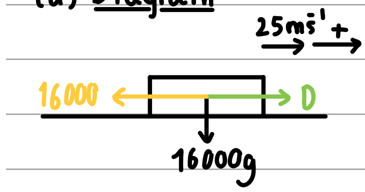
The model for the motion of the lorry along the same road is now refined so that when the speed of the lorry along the same road is V m s^{-1}, the resistance to the motion of the lorry is modelled as having magnitude $640V$ newtons.

Assuming that the engine of the lorry is working at the same rate of 400 kW

(b) use the refined model to find the speed of the lorry when it is accelerating at 2.1 m s^{-2}

(6)

(a) Diagram



we are told it moves with constant speed ∴ use $\Sigma F_x = 0$ M1

$D = 16\,000$ A1

To get Power we will use D in the power formula

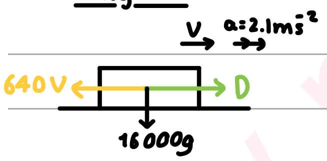
Formula for Power:

Power (W) — $P = Dv$ — Driving force (N), velocity (ms⁻¹)

$P = P$, $D = D = 16\,000\text{N}$, $v = 25\text{ms}^{-1}$ — Substitute: $P = 16000 \times 25$ M1

$P = 400\,000\text{ W}$ — ÷1000 → 400kW shown A1

(b) Diagram



Since it's accelerating use $\Sigma F_x = ma$

$D - 640V = 16000(2.1)$ M1

To get D we will use Power.

Formula for Power:

Power (W) — $P = Dv$ — Driving force (N), velocity (ms⁻¹)

$P = 400\text{kW}$ — ×1000 → $400\,000\text{ W}$, $D = D$, $v = V$ — Substitute: $400\,000 = DV$

$\frac{400\,000}{V} = D$ M1 — substitute this

$\frac{400\,000}{V} - 640V = 16000(2.1)$ A1

$0 = 640V^2 + 33600V - 400\,000$ M1

A1 $0 = 2V^2 + 105V - 1250$ — factorize

$0 = (2V + 125)(V - 10)$

Reject $V = -\frac{125}{2}$, $V = 10$ — Speed is 10ms⁻¹ A1

P 6 1 8 6 9 R A 0 2 1 6

Question 1 continued

P 6 1 8 6 9 R A 0 3 1 6

Question 1 continued

P 6 1 8 6 9 R A 0 4 1 6

Question 1 continued

(Total for Question 1 is 10 marks)

2. Two particles, A and B, of masses $2m$ and $3m$ respectively, are moving on a smooth horizontal plane. The particles are moving in opposite directions towards each other along the same straight line when they collide directly. Immediately before the collision the speed of A is $2u$ and the speed of B is u. In the collision the impulse of A on B has magnitude $5mu$.

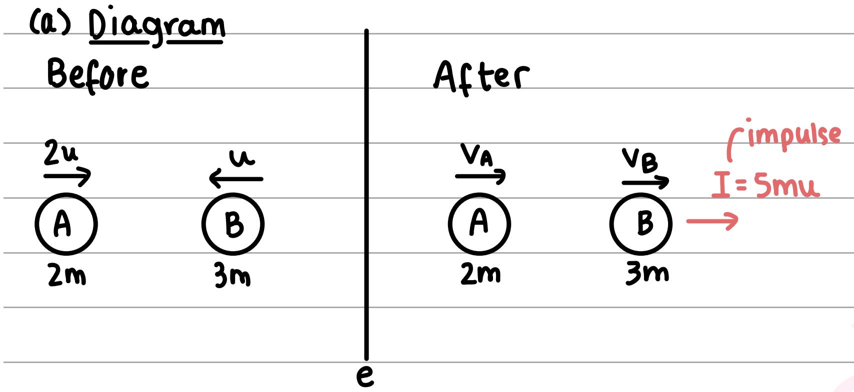
(a) Find the coefficient of restitution between A and B.

(9)

(b) Find the total loss in kinetic energy due to the collision.

(4)

(a) Diagram



Impulse is the change in momentum

Formula for change in momentum:

$$\Delta\text{momentum} = mv_{\text{final}} - mv_{\text{initial}}$$

(mass; initial velocity)

Substitute:

$$I = 3m(V_B - (-u)) = 5mu \quad \text{M1A1}$$

$$3m(V_B + u) = 5mu$$

$$3V_B + 3u = 5u$$

$$V_B = \frac{2u}{3}\ \text{ms}^{-1} \quad \text{A1}$$

We can use the conservation of linear momentum to get V_A M1

conservation of linear momentum means: the total momentum before the collision is the same as the total momentum after.

Formula:

$$m_A u_A + m_B u_B = m_A v_A + m_B v_B$$

(initial velocity; final velocity)

Substitute:

$$2m(2u) + 3m(-u) = 2m(V_A) + 3m(V_B)$$

$$4u - 3u = 2V_A + 3\left(\tfrac{2}{3}u\right) \quad \text{A1}$$

$$u = 2V_A + 2u$$

$$-u = 2V_A$$ the − sign also shows A change direction

$$V_A = -\frac{u}{2}\ \text{ms}^{-1} \quad \text{A1}$$

P 6 1 8 6 9 R A 0 6 1 6

Question 2 continued

We can use Newton's Law of Restitution to get e M1

Newton's Law of Restitution states that: when two objects collide, their speeds after the collision depend on ① speeds before the collision and ② the material from which they're made.

Formula: $e(u_A - u_B) = v_B - v_A$

coefficient of restitution (e), initial speed (u), final speed (v)

Substitute:

$$e(2u - -u) = \frac{2}{3}u - -\frac{u}{2}$$ A1

$$3ue = \frac{7}{6}u$$ cancel u

$$e = \frac{7}{18}$$ value of e A1

(b) To get KE lost: $\Delta KE = KE_I - KE_F$ M1

Formula for Kinetic Energy:

$$KE = \frac{1}{2}mv^2$$

mass (m), velocity (v)

Substitute:

$$\frac{1}{2}(2m)(2u)^2 + \frac{1}{2}(3m)(u)^2 - \left(\frac{1}{2}(2m)\left(-\frac{u}{2}\right)^2 + \frac{1}{2}(3m)\left(\frac{2u}{3}\right)^2\right)$$ A1

$$= 4u^2m + \frac{3}{2}u^2m - \left(\frac{u^2}{4}m + \frac{3}{2}m\left(\frac{4u^2}{9}\right)\right)$$

$$= \frac{11}{2}u^2m - \left(\frac{1}{4}u^2m + \frac{2}{3}u^2m\right)$$ A1

A1 $$= \frac{55u^2m}{12}\ J$$

Units for energy are Joules, J

KE lost.

Turn over

P 6 1 8 6 9 R A 0 7 1 6

Question 2 continued

Question 2 continued

(Total for Question 2 is 13 marks)

3. A particle, P, of mass m kg is projected with speed 5 m s^{-1} down a line of greatest slope of a rough plane. The plane is inclined to the horizontal at an angle α, where $\sin\alpha = \frac{3}{5}$

The total resistance to the motion of P is a force of magnitude $\frac{1}{5}mg$

Use the work-energy principle to find the speed of P at the instant when it has moved a distance 8 m down the plane from the point of projection.

(7)

★ Work-Energy Principle: an increase of KE/GPE/EPE is caused by an equal amount of positive work done on the body (e.g. engine) and a decrease of KE/GPE/EPE is caused by an equal amount of negative work done on the body (e.g. friction).

★ Remember the work-energy formulae:

Either: WD by force + KE$_i$ + GPE$_i$ = KE$_f$ + GPE$_f$ + WD against friction

(work done; initial kinetic; initial grav. potential; final kinetic; final grav. potential; work lost to friction)

OR: WD by force + KE$_i$ + GPE$_i$ − WD by friction = KE$_f$ + GPE$_f$

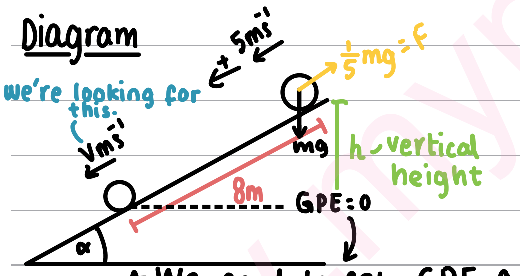
(work done; initial kinetic; initial grav. potential; we subtract this since it leaves the system as heat!; final kinetic; final grav. potential)

★ Formulae for KE and GPE:

$KE = \frac{1}{2}mv^2$ (mass, velocity)

$GPE = mgh$ (mass, change in height, $g = 9.8\text{ms}^{-2}$)

Diagram

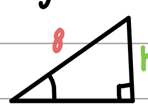


★ We need to set GPE = 0 so we can have a reference point

Substitute:

$\frac{1}{2}m(5)^2 + mgh - 8F = \frac{1}{2}mv^2 + mgh$ (→ 0) M1 M1 B1

To get h:



$\sin\alpha = \frac{3}{5}$, $\sin\alpha = \frac{O}{H} = \frac{h}{8}$

$\therefore \frac{h}{8} = \frac{3}{5}$ $h = \frac{24}{5}$ B1

Sub. h back:

A1 A1 $\frac{25}{2}m + mg\left(\frac{24}{5}\right) - 8\times\left(\frac{1}{5}mg\right) = \frac{1}{2}mv^2$ cancel m's

$2\left(\frac{25}{2} + \frac{24}{5}g - \frac{8}{5}g\right) = v^2$

plug into calculator $2\left(\frac{25}{2} + \frac{16}{5}g\right) = v^2$

$v = 9.37\text{ms}^{-1}$ to 3sf A1

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

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Question 3 continued

(Total for Question 3 is 7 marks)

4. Three particles, P, Q and R, are at rest on a smooth horizontal plane. The particles lie along a straight line with Q between P and R. The particles Q and R have masses m and km respectively, where k is a constant.

Particle Q is projected towards R with speed u and the particles collide directly.

The coefficient of restitution between each pair of particles is e.

(a) Find, in terms of e, the range of values of k for which there is a second collision.

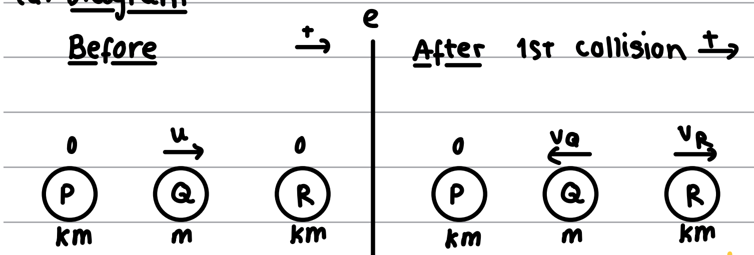
(9)

Given that the mass of P is km and that there is a second collision,

(b) write down, in terms of u, k and e, the speed of Q after this second collision.

(1)

(a) Diagram



for there to be a 2nd collision, Q must change direction so that it hits P, we assume this change in our diagram ∴ our V_Q should end up positive

→ Considering the first collision:

We can use the conservation of linear momentum to get an equation. M1

conservation of linear momentum means: the total momentum before the collision is the same as the total momentum after.

Formula: $m_A u_A + m_B u_B = m_A v_A + m_B v_B$ (u: initial velocity, v: final velocity)

Substitute:

$$m(u) + km(0) = m(-V_Q) + km(V_R)$$

$$mu = -mV_Q + kmV_R \quad \text{cancel } m\text{'s}$$

$$u = -V_Q + kV_R \quad \text{Eq.1} \quad \text{A1}$$

We can use Newton's Law of Restitution to get an equation. M1

Newton's Law of Restitution states that: when two objects collide, their speeds after the collision depend on ① speeds before the collision and ② the material from which they're made.

Formula: $e(u_A - u_B) = v_B - v_A$ (e: coefficient of restitution, u: initial speed, v: final speed)

Substitute:

$$e(u - 0) = V_R - (-V_Q)$$

$$eu = V_R + V_Q \quad \text{Eq.2} \quad \text{A1}$$

P 6 1 8 6 9 R A 0 1 2 1 6

Question 4 continued

Simultaneously solve Eq1 and Eq2: M1

$$u = -V_Q + kV_R \rightarrow \frac{V_Q + u}{k} = V_R$$

$$eu = V_R + V_Q \quad \leftarrow \text{substitute}$$

$$eu = \frac{V_Q + u}{k} + V_Q$$

$$keu = V_Q + u + kV_Q$$

$$keu - u = (1+k)V_Q$$

$$\frac{u(ke-1)}{1+k} = V_Q \quad \text{A1}$$

We want $V_Q > 0$ for the second collision to occur (for V_Q to change direction)

$$V_Q > 0 \quad \text{M1}$$

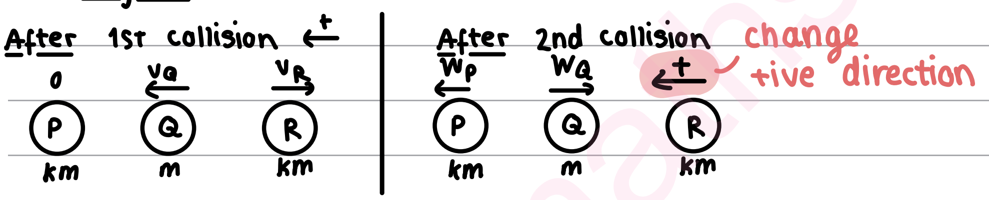
$$\frac{u(ke-1)}{(1+k)} > 0 \quad \text{M1}$$

$$u(ke-1) > 0$$

$$ke - 1 > 0$$

A1 $k > \frac{1}{e}$ Range of values of k

(b) Diagram



The collision is basically the same as that between Q and R.

We see that: $V_Q = U_Q \times \frac{(ke-1)}{(k+1)}$ $(u_Q = u)$

∴ we can deduce that: $W_Q = V_Q \times \frac{(ke-1)}{(k+1)}$

$$W_Q = \frac{u(ke-1)}{k+1} \times \frac{(ke-1)}{(k+1)}$$

B1 $\therefore W_Q = \frac{u(ke-1)^2}{(k+1)^2}$ speed Q after 2^{nd} collision

Question 4 continued

Question 4 continued

Question 4 continued

(Total for Question 4 is 10 marks)

TOTAL FOR FURTHER MECHANICS 1 IS 40 MARKS